INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE (IEB)

SUBJECT ASSESSMENT GUIDELINES

MATHEMATICS

(Updated August 2025)

Table of Contents

A. MEANS OF ASSESSMENT 3
B. REQUIREMENTS 3
C. COGNITIVE LEVEL WEIGHTING ACROSS PAPER 1 AND PAPER 2 4
D. SCHOOL BASED ASSESSMENT 4
E. INTERPRETATION OF REQUIREMENTS 10
F. ADMINISTRATIVE AND SUPPORT DOCUMENTATION 12

A. MEANS OF ASSESSMENT

Paper 1	3 hours	[150]
Paper 2	3 hours	[150]
School Based Assessment (SBA)		[100]

400 marks

B. REQUIREMENTS

Note: An information/ formula sheet will be provided with each examination paper.

MATHEMATICS PAPER 1

Content Weighting	
Description	**Marks**
Bookwork	5 maxima
Algebra and Equations (and inequalities)	25 ± 3
Patterns and Sequences	25 ± 3
Finance, growth and decay	15 ± 3
Functions and Graphs	35 ± 3
Differential Calculus	35 ± 3
Probability	15 ± 3
TOTAL	**150**

Note: Trigonometric graphs will only be examined in Paper 2.

MATHEMATICS PAPER 2

Content Weighting	
Description	**Marks**
Bookwork	6 maxima
Statistics	20 ± 3
Analytical Geometry	40 ± 3
Trigonometry	**50** ± 3
Euclidean Geometry and Measurement	**40** ± 3
TOTAL	**150**

Paper Structure and Design

Questions will be presented in a generally progressive order of difficulty throughout the paper.

- They will not be grouped strictly by topic, meaning easier and more challenging questions from the same topic may appear in different sections of the paper.
- For example, a basic trigonometry question may appear early in the paper, while a more complex, problem-solving trigonometry question could appear later.
- Some questions may contain sub-questions that assess similar content at comparable levels of difficulty.

C. COGNITIVE LEVEL WEIGHTING FOR MATHEMATICS PAPERS 1 AND 2

Assessment tasks are designed to the following cognitive level distribution

Level	Description	%
1	Knowledge	**20 (± 3)**
2	Routine procedures	**30 (± 3)**
3	Complex procedures	**35 (± 3)**
4	Problem solving and investigations – reasoning and reflecting	**15 (± 3)**
	Total	**100**

D. SCHOOL BASED ASSESSMENT (SBA)

SBA constitutes 25% of the total assessment for the National Senior Certificate.

The table below outlines the SBA requirements for Mathematics in Grade 12.

LEARNER FILE REQUIREMENTS – GRADE 12

Task Description	Weighting	Mark
2 Short Items **OR** 1 Long Item	2 × 15 **OR** 1 × 30	30
Three Standardised Tests	3 × 10	30
Grade 12 Preliminary Examination consisting of Paper 1 and Paper 2	2 × 20	40
Total marks:		100

NOTES:

- All work included in the learner file must be completed during the **current academic year**.
- Tasks from **Grade 10 or Grade 11** may **not** be submitted as part of the Grade 12 SBA.
- Schools must retain and, if requested, provide the Grade 12 SBA evidence for moderation by the **IEB** or **Umalusi**.
- These assessment guidelines should be read in conjunction with the **IEB Manual for the Moderation of School-Based Assessment**, available at www.ieb.co.za.

It is strongly recommended that the assessment format in **Grades 10 and 11** align with that of **Grade 12** to promote consistency and support the progressive development of learners' assessment skills.

SHORT ITEMS

Learners are required to complete **two tasks** selected from the list below:

- Guided Discovery
- Skills analysis
- Olympiads
- Investigation
- Computer-Based Tasks*
- Non-traditional Problem solving
- Cheat Sheet
- Modelling*

LONG ITEMS

Learners may complete one long task, which typically requires approximately 5 hours to complete. Some tasks may also include periods of guided contact time in class.

Examples of Long Items:

- Projects: Multi-faceted tasks exploring various mathematical dimensions.
- Discovery Tasks: Substantial pieces of work involving exploration and analysis.
- Computer-Based Investigations: e.g. using Autograph to explore calculus concepts.
- Open-Ended Investigations: Requiring independent thinking and significant effort.
- Mathematical Modelling: Representing and solving real-life problems mathematically.
- Revision Booklets: In-depth review material on key topics (e.g. Functions).
- Comprehensive Formula Sheet: A well-structured summary of key concepts.

STANDARDISED TESTS

Learners must complete **three** standardised tests under controlled conditions:

- Test 1: Focus on content and skills from Paper 1.
- Test 2: Focus on content and skills from Paper 2.
- Test 3: May assess a combination of Paper 1 and/or Paper 2 content.

RELIMINARY EXAMINATIONS

- Learners must complete the **Preliminary Examinations** under **controlled conditions**.
- These papers must follow the **structure, weighting, and format** of the **end-of-year examination for both Paper I and Paper II**.

DETAILED DESCRIPTIONS OF SBA TASKS

1. Translation Task

A translation task tests a learner's ability to convert mathematical notation into language and vice versa. It may also include translating between graphs and equations. This is good practice and should be incorporated across all sections of work. If used as a short task, it should include both aspects of translation.

2. Journal

A journal requires learners to write about and reflect on their own practice. It is a metacognitive activity.

Three possible approaches include:

- The learner identifies questions that posed difficulties and writes notes reflecting on the problem and its solution.
- A teacher-directed journal item in which learners are asked to reflect on a given situation or problem, using written language. For example, the teacher presents an alternate or incorrect solution and asks learners to:
  - comment on the alternate solution (e.g. its strengths and weaknesses), or
  - identify and explain the error, and provide a correct solution.
- The learner reflects on and comments on real-world scenarios requiring mathematical interpretation, such as a magazine or newspaper article.

3. Question Setting

This task can be short (e.g. setting one question) or extended to include multiple components. Learners can design a question or short test, provide a memorandum, write a peer's test, and mark each other's work.

4. Performance (song, dance, speech, and poster)

This is a broad and creative category, often yielding innovative ideas. Posters can be particularly open to misuse, but may also support creative outputs such as cross-number puzzles. More unconventional formats may include skits, raps, or dances that encapsulate a mathematical concept. A poster can also serve as a report on an investigation, describing the problem, method, results, and conclusion.

5. Formula Sheets

This task may be short or extended. As a short task, learners could redesign a formula sheet, justifying their changes. Alternatively, they could be given a selection of formulae and asked to explain their use. For longer tasks, the entire sheet could be analysed, including redesign and example applications of each formula.

6. Teaching a Lesson

This works well as a group or paired activity. Learners prepare and present a portion of content not yet covered in class. Peers and the teacher assess the presentations using clear, openly available criteria.

7. A Lesson to a Friend

In this activity, learners write an explanation of a section of work to a friend. Based on classwork, this allows the learner to reflect on and clarify their own understanding while developing the ability to communicate mathematical ideas in writing.

8. Metacog

A metacog (or mind map) is typically a short task, but may be extended. These tasks should be completed under controlled conditions to authentically reflect the learner's understanding. They reveal the learner's ability to connect ideas and reflect deeply on the topic.
E.g. Create a mind map showing your understanding of the function $f(x) = 2ax^2 - 5x$.

9. Error Spotting

This task may be short or long, depending on its cognitive demands. It may also be incorporated into journal activities.

10. Computer Products

These tasks can take various forms:
- Guided discovery, ideally using tools like Autograph for efficient graphing.
- Independent research using software such as Geometer's Sketchpad.

11. Skills Analysis

This category is broad and can be interpreted in many ways.

- One example involves asking learners to stretch their understanding, e.g. 'Describe how to solve $x/a > 1$.'
- Another involves grappling with new mathematical content and applying it to problems.

12. Non-Traditional Problem Solving

These tasks are varied and open-ended. For example: 'Discuss as many methods as possible to solve the equation ___.' Solutions might involve algebra, graphing, spreadsheet modelling, or trial and error.

13. Olympiads

Learners may enter competitions such as the Harmony Mathematics Olympiad, the UCT Mathematics Competition, or the University of Pretoria Mathematics Competition. Submit both the script and question paper as an SBA item.

14. Cheat Sheet

Only suitable as a short task, this activity requires learners to extract and synthesise key concepts from a topic onto an A5 page. It resembles a mind map or metacog and fosters summary and recall skills.

Note on Tasks 15 to 18:

The following tasks should include some or all of the following steps:

- Identify a problem to be solved
- Make a conjecture following preliminary investigation
- Collect relevant data or information
- Organise and represent data meaningfully
- Draw conclusions
- Refine or finalise a theory
- Document the full process in a report
- Complete within a maximum of three weeks
- Allow for both individual and group assessment

15. Investigation

Investigations may be short or long and should explore a pattern or trend, encouraging testing, conjecture, and conclusion. Topics such as Number Theory and Patterning are rich sources for this kind of task.

16. Discovery (Guided Investigation)

These are structured investigations where learners are guided to arrive at particular conclusions, although further exploration is encouraged.

17. Projects (Multifaceted)

These involve at least three activities across various difficulty levels. Learners must produce a substantial, cohesive piece of work.

18. Modelling

This involves applying mathematics to real-world scenarios.
E.g. *Investigate the claim that citizens in a neighbouring country require a wheelbarrow of money to buy bread. Learners must calculate the volume capacity of a wheelbarrow and compare it to the volume of currency notes to assess the claim's feasibility.* Other examples include analysing how changing the value of a rugby conversion might affect match outcomes.

19. Revision Booklet

This task includes a theory section with worked examples, followed by graded exercises with full solutions. Evidence of metacognitive thinking must be present in the choice of examples, explanations, and reflections included by the learner.

Examples of Mathematics Assessment Tasks Enhanced by AI Integration

Task Type	Description	AI Integration Example
Translation Task	Learners convert between mathematical notation and language, including graphs.	Use AI to generate explanations of algebraic expressions or interpret graphical representations.
Journal	Learners reflect on problem-solving and understanding.	Use AI to provide feedback, compare solutions, or offer prompts for deeper reflection.
Question Setting	Learners design questions and mark schemes.	AI can suggest improvements, generate variations, or provide model answers.
Modelling	Learners explore real-world contexts mathematically.	Use AI to simulate economic/physical models, process real data, or validate feasibility of real-world claims.
Revision Booklet	Learners compile theory and practice into a study resource.	AI helps create worked examples, automate feedback, and personalise revision paths.

E. INTERPRETATION OF REQUIREMENTS

MATHEMATICS
INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1}\ ;\ r \neq 1 \qquad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$A = P(1 + ni) \qquad A = P(1 - ni)$$

$$A = P(1 + i)^n \qquad A = P(1 - i)^n$$

$$F = x\left[\frac{(1+i)^n - 1}{i}\right] \qquad P = x\left[\frac{1 - (1+i)^{-n}}{i}\right]$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}\ ;\ \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.sinC$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\qquad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\qquad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$\bar{x} = \dfrac{\sum fx}{n}$ $\qquad$ $\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \dfrac{n(A)}{n(S)}$ $\qquad$ $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

$\hat{y} = a + bx$ $\qquad$ $b = \dfrac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

F. Administrative And Support Documentation

Appendices

Appendix A: Summary of Candidates' Assessment
A record summarising each learner's SBA components, including task types, marks obtained, and weighting.

Appendix B: Summary of Assessment
A consolidated summary reflecting the performance of the class across different SBA components.

Appendix C: Internal School-Based Assessment Checklist
A checklist used by teachers and heads of departments to ensure that SBA tasks comply with SAGs requirements and assessment standards.

Appendix D: SBA Moderation Form
A moderation tool for internal and external use, confirming that assessment tasks are marked fairly, consistently, and according to agreed criteria.

Appendix E: Letter from the Principal
A formal letter from the school principal certifying that the SBA has been conducted according to national and institutional guidelines.

Appendix F: Learner Declaration – Use of AI (Example)
A sample declaration form to be completed by learners where AI tools were involved in completing any SBA task.

Appendix G: Euclidean Geometry: Acceptable Reasons

Appendix H: Curriculum Content and Clarification
A reference document outlining the prescribed content and skills areas and any specific guidelines or interpretations relevant to the examinations.

APPENDIX A: SUMMARY OF CANDIDATES' ASSESSMENT

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

		Short Items (45 mins +)		**OR**	**Long item (5 hrs.)**		**Tests (45 mins +)**				**Prelims**			**FINAL**
Note: *Please indicate the total marks each assessment was out of.* *Enter the actual marks in the appropriate columns.* *In the* ***Sub-total*** *column, provide the total for that section as a* ***weighted percentage****.* *You may round individual sub-totals for reporting purposes; however, the* ***final overall calculation must be done using unrounded values****.*		Short 1 Out of:	Short 2 Out of:	**Sub-Total**	Long item Out of:	**Sub-Total**	Test 1 Out of	Test 2 Out of	Test 3 Out of	**Sub-Total**	Paper 1 Out of	Paper 2 Out of	**Sub-Total**	
											150	150		
EXAM No.	**Name**	15%	15%	**30%**	30%	**30%**	10%	10%	10%	**30%**	20%	20%	**40%**	**100%**

Declaration by the Candidate's Teacher

I, ____
(*Print name and title of teacher*)

at ____
(*Print name of school*)

hereby declare that the work submitted by these candidates has been **monitored for academic integrity**, including checks for both **plagiarism** and **inappropriate use of Artificial Intelligence (AI)** tools.

I confirm that, to the best of my knowledge, the work represents the candidates' own understanding and effort, and that any support from AI has been critically evaluated and appropriately disclosed by the learners.

Signed (Teacher): ____________________________

Date: ____________________________

APPENDIX B: SUMMARY OF ASSESSMENT

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

NSC Summary of Assessment: Mathematics

This form must be **completed by the candidate** under the **supervision of the teacher**. It should be placed as the **first page** in the learner's Mathematics School-Based Assessment (SBA) file.

Name of candidate: ____

EXAMINATION NUMBER													

Short Items (recommended 45 min)		**Mark**	**Out of**	**Mark as %**
1				
2				
OR				
Long Item (recommended 5 hours)		**Mark**	**Out of**	**Mark as %**
1				
Standardised Tests (recommended 45–60 min)		**Mark**	**Out of**	**Mark as %**
1				
2				
3				
Preliminary Examinations		**Mark**	**Out of**	**Mark as %**
1	**Paper 1**			
2	**Paper 2**			

		Candidate's Marks as %			**Max**	**Final**
Alternative Assessment	**Short OR**				**30**	
	Long				**30**	
Tests	**Formal**				**30**	
Examinations	**Paper 1**				**20**	
	Paper 2				**20**	
FINAL SBA					**100**	

Declaration by the Candidate

I, ____ (*Print full name*) declare that all external sources used in this file have been properly acknowledged and referenced. I confirm that, apart from these sources, all work contained in this file is **my own original work**.

If I made use of **Artificial Intelligence (AI) tools** (e.g., for research support, explanation, or formatting), I have done so **responsibly** and **critically**, ensuring that the final content reflects **my own understanding** and decisions.

I understand that any **misuse of AI**, **plagiarism**, or **misrepresentation of authorship** may result in **disqualification from the National Senior Certificate Examination**.

Signed: ______________________________ **Date:** ______________________

APPENDIX C: INTERNAL SCHOOL BASED ASSESSMENT CHECKLIST

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

School Name: ____

Centre number: ________________________ Number of candidates: ___________

Teachers Names:

1.	Verified that the **summary sheet** is included in the educator's portfolio.	
2.	Ensured that **educator and learner files** contain **no plastic sleeves** or **stapled pages**, and that **file dividers are clearly labelled** in accordance with the summary sheet structure.	
3.	Reviewed the **moderator's report** and confirmed that recommendations have been **carefully considered and implemented** by the relevant teachers.	
4.	Included the **regional moderator's report**, if available (including the previous year's report).	
5.	Where multiple teachers are teaching a grade, ensured that **portfolio tasks and memoranda** are **moderated by a colleague**.	
6.	Confirmed that **internal moderation** has been conducted, and that **standardisation** across classes has occurred (e.g., same question papers, shared markers, and equivalent task difficulty).	
7.	Verified that **all SBA components** are present in the teacher file.	
8.	Included evidence of **internal moderation** (both **pre- and post-assessment**) for each SBA task.	
9.	Monitored the **average results** between **Alternative Assessment** and **Formal Assessment** items—group averages should be within a **10% margin**.	
10.	Confirmed that a **complete set of marks and breakdowns**, including class averages, is present in the teacher's file. Clearly indicate which assessments contribute to each learner's **SBA composite mark**.	
11.	Ensured that **raw scores** are included in the composite mark list and that **rounded marks** are recorded to **two decimal places**.	
12.	Verified that **short items** and **formal tasks** were completed under **controlled conditions**.	
13.	Checked that the **Subject Assessment Guidelines (SAGs) document** is included.	
14.	Provided a **detailed explanation** and, if applicable, a **doctor's note** for any learner absent from an SBA assessment task contributing to the final mark.	

________________________ ________________

HOD: Mathematics Date

APPENDIX D: SBA MODERATION FORM

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

This form is to be used for Regional and National Moderation:

CENTRE	**CENTRE NUMBER**

NUMBER OF EDUCATORS IN THE MATHEMATICS DEPARTMENT		
NAMES OF EDUCATORS		

Type of moderation	Regional	National
	Teacher and Learner	Teacher only

EDUCATORS PORTFOLIO:

ADMINISTRATION

Checklist Criteria form	YES	NO
Letter from Principal	YES	NO
Comment:		
IEB Mark List (downloaded from the IEB online website)	YES	NO
Signed by Principal	YES	NO
Marks entered correctly	YES	NO
Marks correspond with pupil files	YES	NO
Comment:		
Composite Mark List	YES	NO
Correct mark calculation	YES	NO
Comment:		

Important Notes

1. **All portfolio items must show evidence of internal moderation.**

   In cases where only one teacher is responsible for the subject at a school, that teacher is expected to **collaborate with a colleague from another school** to ensure that moderation of tasks is carried out effectively.

2. **Analysis grids must be included for all portfolio items.**

   While it is not required that every task strictly follows the suggested cognitive level distribution, the grids should demonstrate an **awareness of the need to balance cognitive demand** across all four levels of assessment.

ALTERNATIVE ASSESSMENT ITEMS

2 SHORT ITEMS OR 1 LONG ITEM

TYPE:		
Translation	Olympiad with full solution	Guided discovery/Investigation
Error Spotting	Metacog (incl. test)	Other
Other	Other	Other

Short Item 1 (15%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Comment:				
Short Item 2 (15%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Comment:				

LONG ITEM

TYPE:		
Modelling of a real-life situation	Project Multifaceted	Guided discovery/ Investigation
Other	Other	Other

Long Item (30%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Comment:				

FORMAL ASSESSMENT ITEMS

STANDARDISED TESTS

Test 1 (10%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Analysis grid	YES		NO	
Comment:				
Test 2 (10%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Analysis grid	YES		NO	
Comment:				
Test 3 (10%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Analysis grid	YES		NO	
Comment:				
Optional Tests:				

Important Note:

If **more than three tests** are submitted for SBA purposes, it is essential to clearly **indicate each learner's selected tests** on the **composite mark sheet**.

PRELIMINARY EXAMINATION

Paper 1 (20%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Analysis grid	YES		NO	
Comment:				
Paper 2 (20%)	YES		NO	
Levels covered	1	2	3	4
Acceptable length	YES		NO	
Memorandum	YES		NO	
Evidence of internal moderation	YES		NO	
Analysis grid	YES		NO	
Comment:				

COMMENT ON EDUCATORS' PORTFOLIO:

LEARNER PORTFOLIO:

Centre number displayed	YES	NO
Correct file (no plastic sleeves, file dividers)	YES	NO
Copy of learner file mark sheet	YES	NO
Declaration of authenticity	YES	NO
Two short items or One long item	YES	NO
Test 1 covering content from Paper 1	YES	NO
Test 2 covering content from Paper 2	YES	NO
Test 3 covering content from Paper 1 or/and Paper 2	YES	NO
Preliminary exams Paper 1 and Paper 2	YES	NO

COMMENT ON LEARNER PORTFOLIO:

GENERAL COMMENT

MODERATOR NAME: ______________________

SIGNATURE: ______________________

DATE: ______________________

LEARNER PORTFOLIO MODERATION FORM PRIVATE CANDIDATE:

This form is to be used for National Moderation

CENTRE:	CENTRE NUMBER:

LEARNER EXAMINATION NUMBER:	

Centre number displayed	YES	NO
Correct file (No plastic sleeves, file dividers)	YES	NO
Copy of learner file mark sheet	YES	NO
Declaration of authenticity	YES	NO
Two short Items or one long item	YES	NO
Test 1 covering content from Paper 1	YES	NO
Test 2 covering content from Paper 2	YES	NO
Test 3 covering content from Paper 1 or/and Paper 2	YES	NO
Preliminary exams Paper 1 and Paper 2	YES	NO

COMMENT ON LEARNER PORTFOLIO:

____

____

____

____

____

____

____

____

MODERATOR NAME: ________________________

SIGNATURE: ________________________

DATE: ________________________

APPENDIX E: LETTER FROM THE PRINCIPAL

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

SCHOOL: ______________________

ADDRESS: ______________________

____

____

The IEB
PO Box 875
Highlands North
2037

Dear IEB Moderator

RE: SCHOOL BASED ASSESSMENT AND MODERATION OF SBA IN GRADE 12

MATHEMATICS

We hereby certify that the following quality assurance and administrative standards have been upheld:

Teachers of the same subject have met regularly to reflect on and discuss issues of standardisation.	YES	NO
The assessment tasks set for learners are of the required standard.	YES	NO
The memoranda used for marking are accurate and functional.	YES	NO
Learner tasks meet the criteria described in the IEB Subject Assessment Guidelines.	YES	NO
Marking is complete and of an appropriate standard.	YES	NO
All administrative procedures have been correctly followed and completed.	YES	NO
All information on the first page of each learner's file (Appendix B) is complete and correct.	YES	NO

____
TEACHER

DATE: ________________

____
PRINCIPAL

DATE: ________________

APPENDIX F: Learner Declaration – Use of AI in Assessment Tasks

INDEPENDENT EXAMINATIONS BOARD

NATIONAL SENIOR CERTIFICATE EXAMINATION

MATHEMATICS SBA

Candidate Name	
Subject	
Assessment Title	
Teacher	
Assessment Due Date	

I (full name), ____, hereby confirm that:

- I have read and understood my school's policy relating to Academic Dishonesty.
- The task submitted has been completed in full by me.
- I have clearly indicated in the task any sections work that were generated or assisted by AI, giving credit where it is due.
- I have adhered to the school policy regarding Academic Dishonesty – specifically, identifying where AI has been used and citing all sources of information.
- I acknowledge that I may have to verbally explain my answers / solutions presented in this task if asked by my teacher.

Candidate Signature: ____ Date: ______________________________________

Candidate must complete the table below to declare any use of AI tool in their assessment tasks:

Section of Task	**AI Tool Used**	**Purpose of Use**	**Extent of AI Use**	**Candidate's Own Contribution**
e.g. Research, drafting, editing	**e.g. ChatGPT, Grammarly**	**e.g. Idea generation, grammar check**	**e.g. Light, Moderate, Extensive**	**e.g. Wrote all the content, AI checked grammar**

APPENDIX G: EUCLIDEAN GEOMETRY: ACCEPTABLE REASONS

The use of the following shortened versions of the theorem statements is encouraged.

EUCLIDEAN GEOMETRY: ACCEPTABLE REASONS

THEOREM STATEMENT	ACCEPTABLE REASON(S)
LINES	
The adjacent angles on a straight line are supplementary.	∠s on a str line
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj ∠s supp
The adjacent angles in a revolution add up to 360º.	∠s round a pt OR ∠s in a rev
Vertically opposite angles are equal.	vert opp ∠s =
If AB || CD, then the alternate angles are equal.	alt ∠s; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp ∠s; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int ∠s; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠s =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠s =
If the co-interior angles between two lines are supplementary, then the lines are parallel.	coint ∠s supp
TRIANGLES	
The interior angles of a triangle are supplementary.	∠sum in ∠ **OR** sum of ∠s in Δ **OR** Int ∠s Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext ∠of ∠
The angles opposite the equal sides in an isosceles triangle are equal.	∠s opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal ∠s
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	Converse Pythagoras **OR** Converse Theorem of Pythagoras

THEOREM STATEMENT	ACCEPTABLE REASON(S)
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS OR S∠S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS OR ∠∠S
If in two right-angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent	RHS OR 90°HS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2^{nd} side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δs **OR** equiangular Δs
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	Sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height
CIRCLES	
The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius **OR** converse tan ⊥ radius **OR** converse tan ⊥ diameter

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre $\perp$ to chord
The perpendicular bisector of a chord passes through the centre of the circle;	perp bisector of chord
The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	$\angle$ at centre = 2 × $\angle$ at circumference
The angle subtended by the diameter at the circumference of the circle is 90º.	$\angle$s in semi-circle **OR** diameter subtends right angle **OR** $\angle$ in $\frac{1}{2}$ $\odot$
If the angle subtended by a chord at the circumference of the circle is 90º, then the chord is a diameter.	chord subtends 90º **OR** converse $\angle$s in semi-circle
Angles subtended by a chord of the circle, on the same side of the chord, are equal	$\angle$s in the same seg
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal $\angle$s **OR** converse $\angle$s in the same seg
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal $\angle$s
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal $\angle$s
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal $\angle$s
Equal chords in equal circles subtend equal angles at the centre of the circles.	equal circles; equal chords; equal $\angle$s
The opposite angles of a cyclic quadrilateral are supplementary	opp $\angle$s of cyclic quad
If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp $\angle$s quad supp **OR** converse opp $\angle$s of cyclic quad
The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext $\angle$ of cyclic quad
If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext $\angle$ = int opp $\angle$ **OR** converse ext $\angle$ of cyclic quad
Two tangents drawn to a circle from the same point outside the circle are equal in length	Tans from common pt **OR** Tans from same pt

THEOREM STATEMENT	ACCEPTABLE REASON(S)
The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle.	converse tan chord theorem **OR** $\angle$ between line and chord
QUADRILATERALS	
The interior angles of a quadrilateral add up to 360º.	sum of $\angle$s in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are ||
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal , then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp $\angle$s of ||m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp $\angle$s of quad are = **OR** converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** converse diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles	diag of kite

APPENDIX H: CURRICULUM CONTENT AND CLARIFICATION

MATHEMATICS

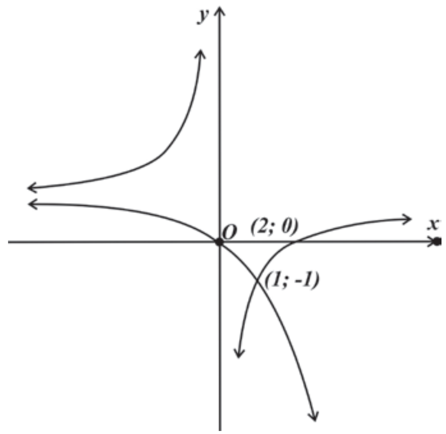
- **Note:** The **sequencing and pacing** of content provided is intended as a **guideline** and may be adapted to suit the specific needs of the learners and teaching context.
- For **examples of cognitive demand** in assessment questions, educators are encouraged to consult the **IEB Analysis Grid**, available at: www.ieb.co.za → *National Senior Certificate* → *Analysis Grid*
- This resource supports alignment with the required cognitive levels and helps ensure balanced assessment design across all content areas.

GRADE 10: TERM 1			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Algebraic expressions**	1. Understand that real numbers can be rational or irrational. 2. Establish between which two integers a given simple surd lies. 3. Round real numbers to an appropriate degree of accuracy. 4. Multiplication of a binomial by a trinomial. 5. Factorisation to include types taught in grade 9 and: • trinomials • grouping in pairs • sum and difference of two cubes 6. Simplification of algebraic fractions using factorisation with denominators of cubes (limited to sum and difference of cubes).	**Examples:** 1. Factorise fully: 1.1 m^2-2m+1 (revision) — Learners must be able to recognise the simplest perfect squares. 1.2 $2x^2-x-3$ — This type is routine and appears in all texts. 1.3 $\frac{y^2}{2}-\frac{13y}{2}+18$ — Learners are required to work with fractions and identify when an expression has been 'fully factorised'. 2. Simplify: $\frac{1-2x}{4x^2-1}-\frac{x+4}{2x^2-3x+1}+\frac{1}{1-x}$

[Adapted from: Curriculum and Assessment Policy Statement (CAPS), Mathematics Grade 10–12, Department: Basic Education © 2011]

2	**Exponents**	1. Revise laws of exponents learnt in Grade 9 where x, $y > 0$ and m, $n \in$ Z: • $x^m \times x^n = x^{m+n}$ • $x^m \div x^n = x^{m-n}$ • $(x^m)^n = x^{mn}$ • $x^m \times y^m = (xy)^m$ Also by definition: • $x^{-n} = \frac{1}{x^n}$; $x \neq 0$, and • $x^0 = 1$, $x \neq 0$ 2. Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for m, $n \in Q$.	**Examples:** 1. Simplify: $(3 \times 5^2)^3 - 75$ 2. Simplify: $\frac{9^x - 1}{3^x + 1}$ 3. Solve for x: 3.1 $2^x = 0{,}125$ 3.2 $2x^{\frac{3}{2}} = 54$ 3.3 $3^{x+1} + 3^{x-1} = \frac{10}{9}$ 3.4 $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$
1	**Numbers and patterns**	Patterns: Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear.	**Examples:** 1. Determine the 5th and the nth terms of the number pattern 10; 7; 4; 1; … (There is an algorithmic approach to answering such questions, $T_n = a + (n-1)d$ is not used in Grade 10.) 2. If the pattern MATHSMATHSMATHS … is continued in this way, what will the 267th letter be? It is not immediately obvious how one should proceed, unless similar questions have been tackled.
2	**Equations and Inequalities**	1. Revise the solution of linear equations. 2. Solve quadratic equations (by factorisation). 3. Solve simultaneous linear equations in two unknowns. 4. Solve word problems involving linear, quadratic or simultaneous linear equations.	**Examples:** 1. Solve for x: $\frac{2x-3}{3} - 3x = \frac{2x}{6}$ 2. Solve for m: $2m^2 - m = 1$ 3. Solve for x and y: $x + 2y = 1$; $\frac{x}{3} + \frac{y}{2} = 1$

		5. Solve literal equations (changing the subject of a formula). 6. Solve linear inequalities and show solution graphically. Use of Interval Notation is required.	4. Solve for *r* in terms of *V*, π and *h*: $V = \pi r^2 h$ 5. Solve for *x*: $-1 \leq 2 - 3x < 8$
3	**Trigonometry**	1. Define the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$, using right-angled triangles. 2. Extend the definitions of $\sin\theta$, $\cos\theta$ and $\tan\theta$, for $0° \leq \theta \leq 360°$. 3. Define the reciprocals of the trigonometric ratios, $\text{cosec}\,\theta$, $\sec\theta$ and $\cot\theta$, using right-angled triangles (these three reciprocals should be examined in grade 10 only) 4. Derive values of the trigonometric ratios for the special cases (without using a calculator) $\theta \in \{0°;30°;45°;60°;90°\}$. 5. Solve two-dimensional problems involving right-angled triangles. 6. Solve simple trigonometric equations for angles between 0° and 90° 7. Use diagrams to determine the numerical values of ratios for angles from 0° to 360°	**Comment:** It is important to stress that: similarity of triangles is fundamental to the trigonometric ratios $\sin\theta$, $\cos\theta$ and $\tan\theta$. **Examples:** 1. If $5\sin\theta + 4 = 0$ and $0° < \theta < 270°$, calculate the value of $\sin^2\theta + \cos^2\theta$ without using a calculator. 2. Trigonometric ratios are independent of the lengths of the sides of a similar right-angled triangle and depend (uniquely) only on the angles, hence we consider them as functions of the angles; and 3. doubling a ratio has a different effect from doubling an angle, for example, generally $2\sin\theta \neq \sin 2\theta$ **Example:** 1. Let *ABCD* be a rectangle, with $AB = 2$ cm. Let *E* be on *AD* such that $A\hat{B}E = 45°$ and $B\hat{E}C = 75°$. Determine the area of the rectangle. 2. Determine the length of the hypotenuse of a right-angled triangle, *ABC*, where $\hat{B} = 90°$, $\hat{A} = 30°$ and $AB = 10$ cm. **Comment:** Solve equation of the form $\sin x = c$, or $a\cos x = c$, or $\tan ax = c$, where *a* and *c* are constants. **Example:** Solve for *x*: $4\sin x - 1 = 3$ for $x \in [0°;90°]$

<table>
<tr><th colspan="4">GRADE 10: TERM 2</th></tr>
<tr><th>No. of weeks</th><th>Topic</th><th>Curriculum statement</th><th>Clarification</th></tr>
<tr><td>4</td><td>Functions</td><td>1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. Note: that the graph defined by $y = x$ should be known from Grade 9.
2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable).
3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0$, $b \neq 1$.</td><td>Comments:
1. A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful.
2. After summaries have been compiled about basic features of prescribed graphs and the effects of parameters a and q have been investigated: a: a vertical stretch (and/or a reflection about the x axis) and q a vertical shift. The following examples might be appropriate:
Examples:
1. Sketched below are graphs of $f(x) = \frac{a}{q}$ and $g(x) = nb^x + t$.
The horizontal asymptote of both graphs is the line y = 1.
Determine the values of a, b, n, q and t.


Remember: that graphs in some practical applications may be either discrete or continuous.</td></tr>
</table>

		4. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ for $\theta \in [0°;360°]$. 5. Study the effect of *a* and *q* on the graphs defined by: $y = a\sin\theta + q$; $y = a\cos\theta + q$; and $y = a\tan\theta + q$ where $a, q \in Q$ for $\theta \in [0°;360°]$. 6. Sketch graphs, find the equations of given graphs and interpret graphs. **Note**: Sketching of the graphs must be based on the observation of number 3 and 5.	**Example:** Sketch the graph defined by $y = -\sin x + \frac{1}{2}$ for $x \in [0°;360°]$. **Note:** Trigonometry graphs will be examined in Paper 2 only.
4	**Euclidean Geometry**	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles. 2. Investigate line segments joining the mid-points of two sides of a triangle. 3. Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Prove these conjectures.	**Comments:** • Triangles are similar if their corresponding angles are equal, or if the ratios of their sides are equal: Triangles *ABC* and *DEF* are similar if $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. They are also similar if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$. • We could define a parallelogram as a quadrilateral with two pairs of opposite sides parallel. Then we investigate and prove that the opposite sides of the parallelogram are equal, opposite angles of a parallelogram are equal, and diagonals of a parallelogram bisect each other. • It must be explained that a single counter example can disprove a Conjecture, but numerous specific examples supporting a conjecture do not constitute a general proof. **Example**: In quadrilateral *KITE*, *KI* = *KE* and *IT* = *ET*. The diagonals intersect at M. Prove that: 1. *IM* = *ME* and 2. *KT* is perpendicular to *IE*. As it is not obvious, first prove that $\Delta KIT \equiv \Delta KET$.

GRADE 10: TERM 3			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Analytical Geometry**	Represent geometric figures on a Cartesian co-ordinate system. Derive and apply for any two points $(x_1;y_1)$ and $(x_2;y_2)$ for the formulae for calculating the: 1. distance between the two points; 2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and 3. coordinates of the mid-point of the line segment joining the two points.	**Example**: Consider the points $P(2;5)$ and $Q(-3;1)$ in the Cartesian plane. 1.1 Calculate the distance *PQ.* 1.2 Find the coordinates of *R* if *M*(–1;0) is the mid-point of *PR.* 1.3 Determine the coordinates of *S* if *PQRS* is a parallelogram. 1.4 Is *PQRS* a rectangle? Why or why not?
2	**Finance and growth**	Use the simple and compound growth formulae $[A = P(1+in)$ and $A = P(1+i)^n]$ to solve problems, including interest, hire purchase, inflation, population growth and other real-life problems. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).	Note: Depreciation should also be taught: $A = P(1-in)$ and $A = P(1-i)^n$
2.5	**Statistics**	1. Revise measures of central tendency in ungrouped data. 2. Measures of central tendency in grouped data: calculation of mean estimate of grouped and ungrouped data and identification of modal interval and interval in which the median lies.	**Comment:** In grade 10, the intervals of grouped data should be given using inequalities, that is, in the form $0 \leq x < 20$ rather than in the form $0-19,\ 20-29,\ \ldots$

3. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, interquartile and semi-interquartile range.
4. Five number summary (maximum, minimum and quartiles) and box and whisker diagram.
5. Use the statistical summaries (measures of central tendency and dispersion), and graphs to analyse and make meaningful comments on the context associated with the given data.

Example:
The mathematics marks of 200 grade 10 learners at a school can be summarised as follows:

Percentage obtained	**Number of candidates**
$0 \leq x < 20$	4
$20 \leq x < 30$	10
$30 \leq x < 40$	37
$40 \leq x < 50$	43
$50 \leq x < 60$	36
$60 \leq x < 70$	26
$70 \leq x < 80$	24
$80 \leq x < 100$	20

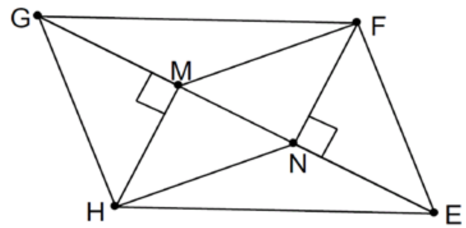
1. Calculate the approximate mean mark for the examination.
2. Identify the interval in which each of the following data items lie:
   2.1 the median
   2.2 the lower quartile
   2.3 the upper quartile
   2.4 the thirtieth percentile

1 **Euclidean Geometry**

Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals.

Comment:
Use congruency and properties of quads, esp. parallelograms. Formal proofs need to be used.

Example:
EFGH is a parallelogram. Prove that *MFNH* is a parallelogram.



2	**Trigonometry**	Problems in two dimensions.	**Example:** Two flagpoles are 30 m apart. The one has height 10 m, while the other has height 15 m. Two tight ropes connect the top of each pole to the foot of the other. At what height above the ground do the two ropes intersect? What if the poles were a different distance apart? (P)
1	**Measurement**	1. Revise the volume and surface areas of right-prisms and cylinders. 2. Study the effect on volume and surface area when multiplying any dimension by a constant factor *k*. 3. Calculate the volume and surface areas of spheres, right pyramids and right cones.	**Example:** The height of a cylinder is 10 cm, and the radius of the circular base is 2 cm. A hemisphere is attached to one end of the cylinder and a cone of height 2 cm to the other end. Calculate the volume and surface area of the solid, correct to the nearest cm^3 and cm^2 respectively. In case of pyramids, bases must either be an equilateral triangle or a square. Problem types must include composite figure.
2	**Probability**	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events *A* and *B* in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{not}(A)) = 1 - P(A)$.	**Comment:** It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. **Example:** A study was done to test how effective three different drugs, A, B and C were in relieving headaches. Over the period covered by the study, 80 patients were given the opportunity to use all two drugs. The following results were obtained: 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 15 reported relief from both drugs A and B 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs 1. Record this information in a Venn diagram. 2. How many subjects got no relief from any of the drugs? 3. How many subjects got relief from drugs A and B, but not C?

			4. What is the probability that a randomly chosen subject got relief from at least one of the drugs? **Comment:** $P(A \text{ or } B)$ is the same as $P(A \cup B)$. $P(A \ and \ B)$ is the same as $P(A \cap B)$.

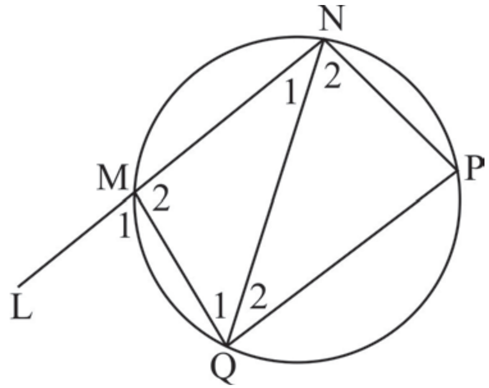
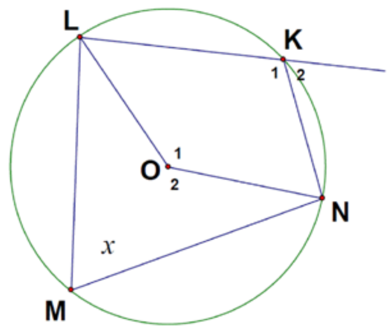
GRADE 11: TERM 1			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Exponents and surds**	1. Simplify expressions using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p}$; $x > 0$; $q > 0$ and including Grade 10 content. 2. Add, subtract, multiply and divide simple surds.	**Examples:** 1. Determine the value of $9^{\frac{3}{2}}$, without the use of a calculator. 2. Simplify: $(3+\sqrt{2})(3-\sqrt{2})$.
3	**Equations and Inequalities**	1. Solve exponential equations and surd equations of the form $\sqrt{x+b} = ax + c,\ a,b,c \in \mathbb{Z}$ 2. Quadratic equations (by factorisation, by completion of the square and by using the quadratic formula). 3. Quadratic inequalities in one unknown (Interpret solutions graphically on number line, and interval notation). NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs. 4. Equations in two unknown, one of which is linear and the other quadratic. 5. Nature of roots.	**Examples:** 1. 1.1 $2^{x+1} = \frac{1}{32}$ 1.2 $x^{\frac{2}{3}} = 4$ 1.3 $\sqrt{x+5} = 3x+1$ 2. 2.1 $x^2 + 2x = 5$ 2.2 $\frac{4}{x^2+4x+3} - \frac{4}{x-2} = \frac{3x+6}{x^2-x-2}$ 3. 3.1 Solve for x : $x^2 \leq 4$ 3.2 Solve for x : $(x+1)(2x-3) \leq 3$ 4. Given $(2x^2+3x-2)(x^2-3) = 0$ Solve for x when: 4.1 $x \in \mathbb{Z}$ 4.2 $x \in \mathbb{Q}$ 4.3 $x \in \mathbb{R}$ 5. Nature of roots. 5.1 Recognition of the types of roots (see example 4). 5.2 Determine the nature of roots and interpret delta/discriminant.

2	**Number patterns**	Patterns: Investigate number patterns including those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	**Examples:** 1. Write down the general term of the sequence: $\frac{1}{2};\frac{4}{5};\frac{9}{10};\frac{16}{17}$ 2. Given the quadratic sequence 4; 9; 17; 28; 42 find the general term.
2.5	**Analytical Geometry**	Derive and apply: 1. the equation of a line though two given points; 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. the inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line $(0^\circ \leq \theta < 180^\circ)$.	**Example:** Given the points $A(2;5)$; $B(-3;-4)$ and $C(4;-2)$, determine: 1. the equation of the line AB; and 2. the size of $B\hat{A}C$.

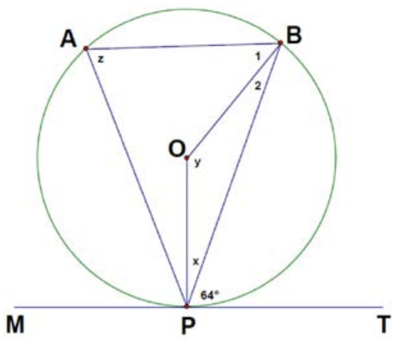
GRADE 11: TERM 2			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
4	**Functions**	1. Revise the effect of the parameters *a* and *q* and investigate the effect of *p* on the graphs of the functions defined by: 1.1 $y = f(x) = a(x+p)^2 + q$ 1.2 $y = f(x) = \frac{a}{x+p} + q$ 1.3 $y = f(x) = ab^{x+p} + q$ where $b > 0, b \neq 1$ 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. 3. Investigate the effect of the parameter *k* on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$. 4. Investigate the effect of the parameter *p* on the graphs of the functions defined by $y = \sin(x+p)$, $y = \cos(x+p)$ and $y = \tan(x+p)$. 5. Draw sketch graphs defined by: $y = a\sin k(x+p)$, $y = a\cos k(x+p)$ and $y = \tan k(x+p)$ at most two parameters at a time.	**Comments:** • Once the effects of the parameters have been established, various problems need to be set: drawing sketch graphs, determining the defining equations of functions from sufficient data, making deductions from graphs. Real life applications of the prescribed functions should be studied. • Two parameters (maximum) at a time can be varied in tests or examinations. **Example**: (To be assessed in Paper 2 only.) Sketch the graphs defined by $y = -\frac{1}{2}\sin(x + 30°)$ and $f(x) = \cos(2x - 120°)$ on the same set of axes, where $-360° \leq x \leq 360°$.

4	**Trigonometry**	1. Derive and use the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\theta \neq k.90°$, k an odd integer and $\sin^2\theta + \cos^2\theta = 1$. 2. Derive and use reduction formulae to simplify the following expressions: 2.1 $\sin(90° \pm \theta);\cos(90° \pm \theta)$; 2.2 $\sin(180° \pm \theta);\cos(180° \pm \theta)$; $\tan(180° \pm \theta)$; 2.3 $\sin(360° \pm \theta);\cos(360° \pm \theta)$; $\tan(360° \pm \theta)$; and 2.4 $\sin(-\theta)$; $\cos(-\theta)$; $\tan(-\theta)$ 3. Determine for which values of a variable an identity holds. 4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals.	**Comment:** Teachers should explain where reduction formulae come from. **Examples:** 1. Prove that $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$. 2. For which values of θ is $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$ undefined. 3. Simplify $\frac{\cos(180° - x)\sin(x - 90°) - 1}{\tan^2(540° + x)\sin(90° x)\cos(-x)}$. 4. Determine the general solution of $\cos^2\theta + 3\sin\theta = -3$.

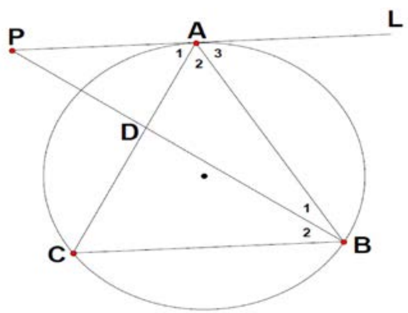
GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
1	**Measurement**	1. Revise the Grade 10 work.	Formulae for right prisms and cylinders will not be given in examinations.
3	**Euclidean Geometry**	Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact. Then investigate and prove the theorems of the geometry of circles: • The line drawn from the centre of a circle perpendicular to a chord bisects the chord; • The perpendicular bisector of a chord passes through the centre of the circle; • The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); • Angles subtended by a chord of the circle, on the same side of the chord, are equal; • The opposite angles of a cyclic quadrilateral are supplementary; • Two tangents drawn to a circle from the same point outside the circle are equal in length;	**Comments:** Proofs of the following theorems (acute angle case only) are examinable, their converses (where they exist) are not: Chords in circles • Line through centre and midpoint Angles in circles • Angle at centre = 2 × angle at circumference Cyclic Quadrilaterals • Opposite angles of cyclic quad Tangents to circles • Tan-chord theorem Use as results: • Angle subtended by a diameter is 90°. • Exterior angle of a cyclic quadrilateral is equal to int. opposite angle. • Angles in same segment are equal. • Two tangents drawn from same point outside a circle are equal. • The radius is perpendicular to the tangent at the point of contact. **Also:** • Diagrams for proofs will always be given. • Riders will place emphasis on proof, e.g. prove $x = 20°$; prove that $\hat{D} = 2x + y$; prove $AB \parallel CD$; prove $ABCD$ is a cyclic quad; name 4 other angles equal to x. • NO concurrency and NO proof by contradiction.

		• The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. Use the above theorems and their converses, where they exist, to solve riders.	**Examples:** 1.  Given: $\hat{N}_1 = 35°$ $\hat{N}_2 = 45°$ $\hat{Q}_1 = 50°$ (a) Write down, with a reason, the size of $\hat{M}_1$ (b) Prove $MN = NP$ 2. O is the centre of the circle above and $\hat{O}_1 = 2x$.  2.1 Determine $\hat{O}_2$ and $\hat{M}$ in terms of *x*. 2.2 Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of *x*. 2.3 Determine $\hat{K}_1$ + $\hat{M}_2$. What do you notice? 2.4 Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$.

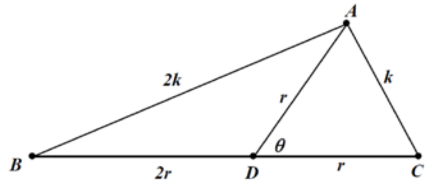
3. O is the centre of the circle above and MPT is a tangent. Also, $OP \perp MT$.
Determine, with reasons, x, y and z.



4. Given: $AB = AC$, $AP \parallel BC$ and $\hat{A}_2 = \hat{B}_2$.



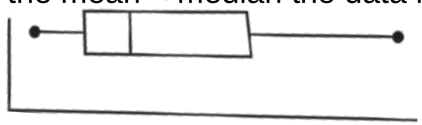
Prove that:
4.1 PAL is a tangent to circle ABC,
4.2 AB is a tangent to circle ADP.

2	**Trigonometry**	1. Prove and apply the sine, cosine and area rules. 2. Solve problem in two dimensions using the sine, cosine and area rules.	**Comment:** The proofs of the sine, cosine and area rules are examinable. The proofs will be assessed in acute angle triangles only. The area rule may not assume the sine rule and vice versa. **Example:** In ΔABC, D is on BC, $A\hat{D}C = \theta$, $DA = DC = r$, $BD = 2r$, $AC = k$ and $BA = 2k$.  Show that $\cos\theta = \dfrac{1}{4}$.
2	**Finance, growth and decay**	1. Use simple and compound decay formulae: $A = P(1 - in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). 2. The effect of different periods of compound growth and decay, including nominal and effective interest rates.	**Examples:** 1. The value of a piece of equipment depreciates from R10 000 to R5 000 in four years. What is the rate of depreciation if calculated on the: 1.1 straight line method; and 1.2 reducing balance? 2. Which is the better investment over a year or longer: 10,5% p.a. compounded daily or 10,55% p.a. compounded monthly? **Comment:** The use of a timeline to solve problems is a useful technique. 3. R50 000 is invested in an account which offers 8% p.a. interest compounded quarterly for the first 18 months. The interest then changes to 6% p.a. compounded monthly. Two years after the money is invested, R10 000 is withdrawn. How much will be in the account after 4 years? **Comment:** Stress the importance of not working with rounded answers, but of using the maximum accuracy afforded by the calculator right to the final answer when rounding might be appropriate.

<table>
<tr><th colspan="4">GRADE 11: TERM 4</th></tr>
<tr><th>No. of weeks</th><th>Topic</th><th>Curriculum statement</th><th>Clarification</th></tr>
<tr>
<td>3</td>
<td>Statistics</td>
<td>1. Histograms
2. Frequency polygons
3. Ogives (cumulative frequency curves)
4. Variance and standard deviation of ungrouped data
5. Symmetric and skewed data
6. Identification of outliers</td>
<td>Comments:
• Variance and standard deviation may be calculated using calculators.
• Problems should cover topics related to health, social, economic, cultural, political and environmental issues.
• Symmetry and skewness should be done in context of a box and whisker diagram and histogram by observation, i.e. if values on one side tend to extend and 'tail off'. Also by comparing values of mean and median.
• Normal distribution will not be examined.

Examples:
1. Consider the following statistics summary:
<table>
<tr><td>n</td><td>Mean</td><td>Median</td><td>σ</td><td>Minimum</td><td>Maximum</td><td>Q_1</td><td>Q_3</td></tr>
<tr><td>48</td><td>68,35</td><td>69,90</td><td>10,20</td><td>43,20</td><td>87,40</td><td>59,15</td><td>74,75</td></tr>
</table>
(a) Draw the box and whisker plot for the data summarised in the table.
(b) Would you describe the distribution of this data as skewed or symmetric? If skewed, in what direction? Explain your answer.
(c) If an outlier is a value of greater than $Q_3 + 1{,}5 \times IQR$ or less than $Q_1 - 1{,}5 \times IQR$, where IQR is the interquartile range, show that there are no outliers in this data set.
NB: Learners are expected to comment to the relationship between median and mean when discussing skewness.</td>
</tr>
</table>

2. Choose the correct statement:
   (a) the data is positively skewed
   (b) the mean < median the data is skewed to the left
   (c)

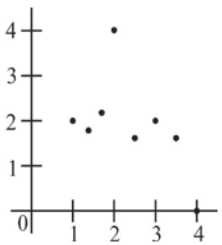


Comment:

- Outliers detection is important for effective modelling. Outliers should be excluded from such model fitting.
- Identification of outliers should be done in the context of a scatter plot as well as the box and whisker diagrams. Learners are not expected to memorise formulas for determining outliers.

Examples:

1. Consider the scatter plot drawn and answer the questions that follow.



   (a) Write down the co-ordinates of two points that are outliers.
   (b) Draw in a line of best fit.

Example:

An outlier is any value that lies more than one and a half times the length of the box from either end of the box.
That is, a data value is an outlier if it is less than Q_1 + 1,5 × IQR or greater than Q_3 + 1,5 × IQR
Where Q_1 is the lower quartile, Q_3 is the upper quartile and IQR is the interquartile range.
Find the outliers, if any for the following data set:
10 14 14 15 15 15 16 18

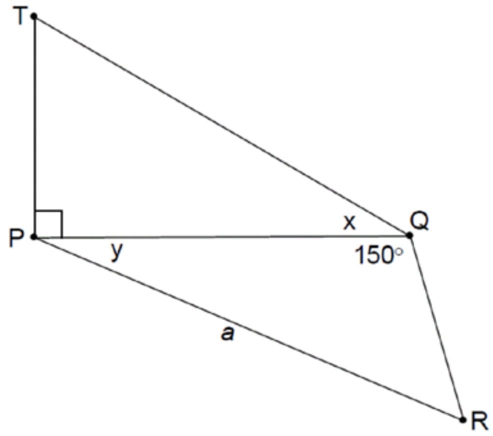
2	**Probability**	1. Revise and use tree diagrams and Venn diagrams to solve probability problems. 2. The use of tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent. 3. Revise the addition rule for mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$, the complementary rule: $P(\text{not } A) = 1 - P(A)$ and the identity $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. 4. Dependent and independent events and the product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$	**Examples:** 1. $P(A)=0,45, P(B)=0,3$ and $P(A \text{ or } B)=0,615$. Are the events A and B mutually exclusive, independent or neither mutually exclusive nor independent? 2. What is the probability of throwing at least one six in four rolls of a regular six-sided die? **Comment:** Venn Diagrams or Contingency tables can be used to study dependent and independent events. **Example:** In a group of 50 learners, 35 take Mathematics and 30 take History, while 12 take neither of the two. If a learner is chosen at random from this group, what is the probability that he/she takes both Mathematics and History? **Comment** $P(A \text{ or } B)$ is the same as $P(A \cup B)$. $P(A \text{ and } B)$ is the same as $P(A \cap B)$.

GRADE 12: TERM 1			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Patterns, sequences, series**	1. Number patterns, including arithmetic and geometric sequences and series 2. Sigma notation 3. Derivation and application of the formulae for the sum of arithmetic and geometric series: 3.1 $S_n = \frac{n}{2}[2a + (n-1)d]$; $S_n = \frac{n}{2}(a + l)$ 3.2 $S_n = \frac{a(r^n - 1)}{r - 1}$; $(r \neq 1)$; and 3.3 $S_\infty = \frac{a}{1-r}$; $(-1 < r < 1)(r \neq 1)$	**Comment:** Derivation of the formulae is examinable. Questions will be asked in such a way that tests understanding of the proof rather than memorisation of the proof. **Example:** An arithmetic sequence has a first term *a* and the common difference *d.* A student adds the first *n* terms by writing the sequence forward and the gain backwards, as shown below: $S_n = a + (a+d) + (a+2d) + \cdots + [a + (n-1)d]$ $S_n = [a + (n-1)d] + [a + (n-2)d] + \cdots + a$ (a) Explain why adding the two sequences term-by-term gives $2S_n = n[2a + (n-1)d]$ (b) Hence, determine S_n. **Examples:** 1. 1.1 Write down the first terms of the sequence with general term $T_k = \frac{1}{3k-1}$ 1.2 $\sum_{k=0}^{3}(3k-1)$ 2. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. 3. Determine the largest value of n such that $\sum_{i=1}^{n}(3i-2) < 2000$ 4. Show that 0,9 = 1

3	**Functions**	1. Definition of a *function.* 2. General concept of the *inverse of a function* and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function. 3. Determine the sketch graphs of the inverses of the functions defined by $y = ax + q$; $y = ax^2$ $y = b^x$; $(b > 0, b \neq 1)$	**Examples:** 1. Consider the function *f* where $f(x) = 3x - 1$. 1.1 Write down the domain and range of *f.* 1.2 Show that *f* is a one-to-one relation. 1.3 Determine the inverse function f^{-1}. 1.4 Sketch the graphs of the functions f, f^{-1} and $y = x$ line on the same set of axes. 2. Repeat Question 1 for the function $f(x) = -x^2$ and $x \leq 0$.
		Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases/ decreases.	**Comments:** 1. Do not confuse the inverse function $f^{-1}(x)$ with the reciprocal $\frac{1}{f(x)}$. For example, for the function where $f(x) = \sqrt{x}$, the reciprocal is $\frac{1}{\sqrt{x}}$, while $f^{-1}(x) = x^2$ for $x \geq 0$. 2. Note that the notation $f^{-1}(x) = \ldots$ is used only for one-to-one relation and must not be used for inverses of many-to-one relations, since in these cases the inverses are not functions.
1	**Functions: exponential and logarithmic**	1. Revision of the exponential function and the exponential laws and graph of the function defined by $y = b^x$ where $b > 0$ and $b \neq 1$. 2. Understand the definition of a logarithm: $y = \log_b x \Leftrightarrow x = b^y$, where b > 0 and $b \neq 1$. 3. The graph of the function define $y = \log_b x$ for both the cases $0 < b < 1$ and b > 1.	**Comments:** The four logarithmic laws will be applied in basic situations. Questions will not require the use of more than one law in any one question. $\log_b(AB) = \log_b A + \log_b B$ $\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$ $\log A^n = n\log A$; and $\log_B A = \frac{\log A}{\log B}$ They follow from the basic exponential laws of grade 10.

			Caution Make sure learners know the difference between the two functions defined by $y = b^x$ and $y = x^b$ where *b* is a positive (constant) real number. **Examples:** 1. Solve for *x*: $75\,(1{,}025)^{x-1} = 300$ $\quad$ $75\,(1{,}025)^{x-1} = 300$ 2. Let $f(x) = a^x$, a > 0. 2.1 Determine *a* if the graph of *f* goes through the point $(2;\frac{25}{16})$. 2.2 Determine the function f^{-1}. 2.3 For which values of *x* is $f^{-1}(x) > -1$? 2.4 Determine the function *h* if the graph of *h* is the reflection of the graph of *f* through the *y*-axis. 2.5 Determine the function *k* if the graph of *k* is the reflection of the graph of *f* through the *x*-axis. 2.6 Determine the function *p* if the graph of *p* is obtained by shifting the graph of *f* two units to the left. 2.7 Write down the domain and range for each of the functions *f*, f^{-1}, *h*, *k* and *p*. 2.8 Represent all these functions graphically.
2	**Finance, growth and decay**	1. Solve problems involving present value and future value annuities. 2. Make use of logarithms to calculate the value of *n*, the time period, in the equations $A = P(1+i)^n$ or $A = P(1-i)^n$. 3. Critically analyse investment and loan options and make informed decisions as to best option(s) (including pyramid schemes).	**Comment:** 1. Derivation of the formulae for present and future values using the geometric series formula $Sn = \frac{a(r^n - 1)}{r-1}; r \neq 1$, will not be required for examination purposes, but should be part of the teaching process to ensure that the learners understand where the formulae come from. The two annuity formulae: $F = \frac{x((1+i)^n - 1)}{i}$ and $P = \frac{x(1-(1+i)^{-n})}{i}$ hold only when payment commences one period from the present and ends after n periods. 2. **NB**. No variations of the above formulae will be examinable. 3. The use of a timeline to analyse problems is a useful technique.

			Examples: 1. Given that a population increased from 120 000 to 214 000 in 10 years, at what annual (compound) rate did the population grow? 2. In order to buy a car, John takes out a loan of R25 000 from the bank. The bank charges an annual interest rate of 11% p.a. compounded monthly. The instalments start a month after he has received the money from the bank. 2.1 Calculate his monthly instalments if he has to pay back the loan over a period of 5 years. 2.2 Calculate the outstanding balance of his loan after two years (immediately after the 24th instalment).
2	**Trigono-metry**	Compound angle identities: $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ $\sin(a \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\sin 2\alpha = 2\sin\alpha\cos\alpha$ $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ $\cos 2\alpha = 2\cos^2\alpha - 1$ $\cos 2\alpha = 1 - 2\sin^2\alpha$	**Comment**: The derivation of the compound and double angle formulae will not be required for examination purposes, but should be part of the teaching process. **Examples:** 1. Determine the general solution of $\sin 2x + \cos x = 0$. 2. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$. 3. Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

GRADE 12: TERM 2			
No. of weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Trigonometry**	1. Solve problems in two and three dimensions.	**Examples:**  1. *TP* is a tower. Its foot, *P*, and the points *Q* and *R* are on the same horizontal plane. From *Q* the angle of elevation to the top of the building is *x*. Furthermore, $P\hat{Q}R = 150°$, $Q\hat{P}R = y$ and the distance between *P* and *R* is *a* metres. Prove that $TP = a\tan x(\cos y - \sqrt{3}\sin y)$ 2. In ΔABC, $AD \perp BC$. Prove that: (a) $a = b\cos C + c\cos B$ where $a = BC$; $b = AC$ and $c = AB$. (b) $\dfrac{\cos B}{\cos C} = \dfrac{c - b\cos A}{b - c\cos A}$ (on condition that $\hat{C} \neq 90°$). (c) $\tan A = \dfrac{a\sin C}{b - a\cos C}$ (on condition that $\hat{A} \neq 90°$). (d) $a + b + c = (b + c)\cos A + (c + a)\cos B + (a + b)\cos C$.

1	**Functions: Polynomials**	Factorise third-degree polynomials. Apply the remainder and Factor Theorems to polynomials of degree at most 3 (no proofs required).	**Comment**: Any method may be used to factorise third degree polynomials in the examinations, but the teaching process should include examples which require the Factor Theorem. **Example:** Solve for *x*: $x^3 + 8x^2 + 17x + 10 = 0$ 1. Solve for *x*: $x^3 + 8x^2 + 17x + 10 = 0$ 2. If $x - 2$ is a factor of $x^3 + px + 6$, determine the value of *p*.
3	**Differential Calculus**	1. An intuitive understanding of the limit concept, in the context of approximating the rate of change or gradient of a function at a point. 2. Use limits to define the derivative of a function *f* at any *x*: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ Generalise to find the derivative of *f* at any point *x* in the domain of *f* i.e. define the derivative function $f'(x)$ of the function $f(x)$. Understand intuitively that $f'(a)$ is the gradient of the tangent to the graph of *f* at the point with *x*-coordinate *a*. 3. Using the definition (first principle), find the derivative, $f'(x)$ for *a*, *b* and *c* constants: (a) $f(x) = ax^2 + bx + c$; (b) $f(x) = ax^3$;	**Comment:** Differentiation from first principles will be examined on any of the types described in 3 (a), (b) and (c) in the 'Curriculum statement'. **Examples:** 1. In each of the following cases, find the derivative of $f(x)$ at the point where $x = -1$, using the definition of the derivative: 1.1 $f(x) = x^2 + 2$ 1.2 $f(x) = \frac{1}{2}x^2 + x - 2$ 1.3 $f(x) = -x^3$ 1.4 $f(x) = -\frac{2}{x}$ **Caution**: Care should be taken not to apply the sum rule for differentiation (4(a)) in a similar way to products. (a) Determine $\frac{d}{dx}((x+1)(x-1))$. (b) Determine $\frac{d}{dx}(x+1) \times \frac{d}{dx}(x-1)$. (c) Write down your observation.

<table>
<tr>
<td></td>
<td></td>
<td>
(c) $f(x)=\frac{a}{x}$;
(d) $f(x)=c$

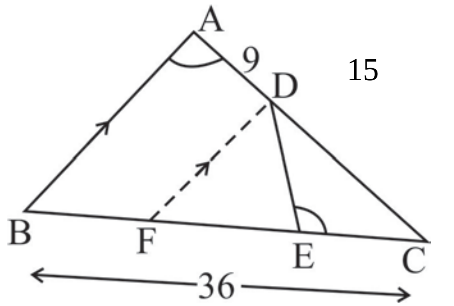
4. Use the formula $\frac{d}{dx}(ax^n)=anx^{n-1}$, (for any real number n) together with the rules:
(a) $\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$ and
(b) $\frac{d}{dx}[kf(x)]=k\frac{d}{dx}[f(x)]$ (k a constant)
5. Find equations of tangents to graphs of functions.
6. Introduce the second derivative $f''(x)=\frac{d}{dx}(f'(x))$ of $f(x)$ and how it determines the concavity of a function.
7. Sketch graphs of cubic polynomial functions using differentiation to determine the co-ordinate of stationary points, and points of inflection (where concavity changes). Also, determine the x-intercepts of the graph using the factor theorem and other techniques.
</td>
<td>
2. Use differentiation rules to do the following:
2.1 Determine $f'(x)$ if $f(x)=(x+2)^2$
2.2 Determine $f'(x)$ if $f(x)=\frac{(x+2)^3}{\sqrt{x}}$
2.3 Determine $\frac{dy}{dt}$ if $y=\frac{t^2-1}{2t+2}$
2.4 Determine $f'(\theta)$ if $f(\theta)=(\theta^{3/2}-3\theta^{-1/2})^2$

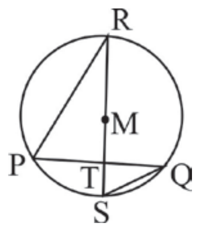
3. Determine the equation of the tangent to the graph defined by $y=(2x+1)^2(x+2)$ where $x=\frac{3}{4}$.
4. Sketch the graph defined by $y=-x^3+4x^2-x$ by:
4.1 finding the intercepts with the axes;
4.2 finding maxima, minima and the co-ordinates of the point of inflection;
(Remember: To understand points of inflection, an understanding of concavity is necessary. This is where the second derivative plays a role.)

5. The radius of the base of a circular cylindrical can is x cm, and its volume is 430 cm^3.
5.1 Determine the height of the can in terms of x.
5.2 Determine the area of the material needed to manufacture the can (that is, determine the total surface area of the can) in terms of x.
5.3 Determine the value of x for which the least amount of material is needed.
5.4 If the cost of the material is R500 per m^2, what is the cost of the cheapest can (labour excluded)? to manufacture such a can.
</td>
</tr>
</table>

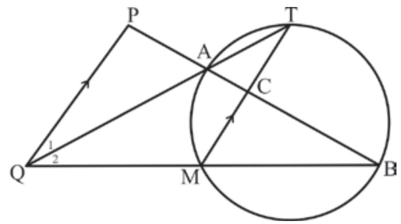
		8. Solve practical problems concerning optimisation and rate of change, including calculus of motion.	6. The motion of a particle is described by the function $S(t) = st^2 - 3t$. 6.1 Find an expression for the velocity of the particle. 6.2 Determine the acceleration of the particle.
2	**Analytical Geometry**	1. The equation $(x-a)^2 + (y-b)^2 = r^2$ defines a circle with radius *r* and centre (*a*;*b*). 2. Determination of the equation of a tangent to a given circle.	**Examples:** 1. Determine the equation of the circle with centre (–1;2) and radius $\sqrt{6}$. 2. Determine the equation of the circle which has the line segment with endpoints (5;3) and (–3;6) as diameter. 3. Determine the equation of a circle with a radius of 6 units, which intersects the *x*-axis at (–2;0) and the *y*-axis at (0;3). How many such circles are there? 4. Determine the equation of the tangent that touches the circle defined by $x^2 - 2x + y^2 + 4y = 5$ at the point (–2;–1). 5. The line with the equation $y = x + 2$ intersects the circle defined by $x^2 + y^2 = 20$ at *A* and *B*. 5.1 Determine the co-ordinates of *A* and *B*. 5.2 Determine the length of chord *AB*. 5.3 Determine the co-ordinates of *M*, the midpoint of *AB*. 5.4 Show that $OM \perp AB$ where *O* is the origin. 5.5 Determine the equations of the tangents to the circle at the points *A* and *B*. 5.6 Determine the co-ordinates of the point C where the two tangents in (5.5) intersect. 5.7 Verify that *CA* = *CB.* 5.8 Determine the equations of the two tangents to the circle, both parallel to the line with the equation *y* = –2*x* + 4. 6. Determine the length of the tangent drawn from A(–2;5) to the circle with equation $x^2 + (y-10)^2 = 4$. 7. Given the circles: $x^2 + y^2 = 1$ and $(x-3)^2 + (y-4)^2 = 16$ Show that the circles touch each other.

<table>
<tr><th colspan="4">GRADE 12: TERM 3</th></tr>
<tr><th>No. of weeks</th><th>Topic</th><th>Curriculum statement</th><th>Clarification</th></tr>
<tr><td>2</td><td>Euclidean Geometry</td><td>1. Revise earlier work on the necessary and sufficient conditions for polygons to be similar.
2. Prove (accepting results established in earlier grades):
• that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem);
• that equiangular triangles are similar;
• that triangles with sides in proportion are similar; and
• the Pythagorean Theorem by similar triangles</td><td>Comments:
• Riders will concentrate on proof of the following types:
(a) Prove $\Delta ABC \mid\mid\mid \Delta DEF$
(b) Prove $AB.PR = AC.PQ$ (i.e. must know to rearrange and identify which triangles to prove similar)
• Riders will also concentrate on numerical calculations.
• Combinations of Grade 12 with Grade 11 Geometry is examinable.
• Constructions may be required to be drawn in order to answer questions.

Examples:
1. 1.1 Prove that $\Delta CDE \mid\mid\mid \Delta CBA$
1.2 Calculate
(a) EC
(b) CF
(c) FE


2. 

M is the circle centre
$RMS \perp PQ$
$PQ = 4x$; $TS = x$ and $RT = 150$ mm</td></tr>
</table>

			 3. 3.1 If $\hat{Q}_1 = x$ name two other angles equal to, giving reasons. 3.2 Prove that $\Delta PBQ \vert\vert\vert \Delta PQA$ 3.3 Deduce that $PQ = \sqrt{PA.PB}$ 3.4 If PB = 11,25 cm; QM = 9 cm and MB = 6 cm calculate the length of CB. **Comment:** **The following proofs are examinable:** Proportion • Line parallel to one side of triangle Similar Triangles • Triangles equiangular • Perpendicular from right angle to hypotenuse

<table>
<tr>
<td>1</td>
<td>Statistics (regression and correlation)</td>
<td>1. Revise symmetric and skewed data.
2. Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.</td>
<td>

Comments:

- Ability to suggest a function type of best fit by inspection, but find least squares regression line $y = a + bx$ using technology (calculator).
- Know that $(\bar{x};\bar{y})$ lies on line of best fit.
- Identify the correlation coefficient (r) as the value that quantifies the strength and direction of the linear relationship between the variables in a set of bivariate data. Interpretation of r values between $-1 \leq r \leq 1$
- Beware: Correlation does not imply causation.

Examples:

1. The following table summarises the number of revolutions x (per minute) and the corresponding power output y (horse power) of a Diesel engine:

x	400	500	600	700	750
y	580	1030	1420	1880	2100

1.1 Find the least squares regression line $y = a + bx$
1.2 Use this line to estimate the power output when the engine runs at 800 m.
1.3 Roughly how fast is the engine running when it has an output of 1200 horse power?

2. An r value for a certain set of bivariate is calculated to have a value equal to –0,243. (There may be more than one.) Select the correct interpretation(s) of the value r.

2.1 a strong positive relationship
2.2 a weak relationship
2.3 a moderate negative relationship
2.4 an indication that as one variable increases the other decreases
2.5 a strong negative relationship
2.6 no relationship

</td>
</tr>
</table>

2	**Counting and probability**	1. Revise: • the concept of **sample space S** as the set of all possible outcomes in a probability experiment; • dependent and independent events; • the product rule for independent events: $P(A \text{ and } B) = P(A) \times P(B)$. • the sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$ • the identity: P(A U B) = P(A or B). $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$ • the complementary rule: $P(\text{not } A) = 1 – P(A)$ 2. Probability problems using Venn diagrams, trees, two-way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent). 3. Apply the fundamental counting principle to solve probability problems.	**Examples:** 1. Given $P(A) = 0{,}4$; $P(B) = 0{,}25$ and $P(A \cap B) = 0{,}1$ 1.1 Determine if A and B are independent events. 1.2 Evaluate $P(A \cup B)$ 2. How many three-character codes can be formed if the first character must be a letter and the second two characters must be different digits? 2.1 if repetition is allowed 2.2 if repetition is not allowed 3. A flag comprises 5 vertical bands  Bands are available in 7 colours. Determine the number of different flags that can be created if: 3.1 Each band is a different colour. 3.2 The first, third and fifth bands are the same colour. 4. What is the probability that a random arrangement of the letters BAFANA starts and ends with an 'A'? 5. Four different glasses and 5 different bottles are arranged on a shelf. How many arrangements are there if they are placed 5.1 at random? 5.2 all the glasses together and all bottles together? in alternating positions? 6. Four red discs, 2 blue discs and 5 yellow discs are places in a bag. 2 discs are randomly chosen without replacement. Find the probability that: 6.1 both are red 6.2 both are the same colour 6.3 both are not blue 6.4 you get one red and one blue disc **Comment:** Questions needing permutations or combinations are not examinable.